Theory of Plasma Contactors in Ground-Based Experiments and Low Earth Orbit

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Previous theoretical work on plasma contactors as current collectors has fallen into two categories: collisionless, double-layer theory (describing space charge limited contactor clouds) and collisional quasineutral theory. Ground-based experiments at low current are well explained by double-layer theory, but this theory does not scale well to power generation by electrodynamic tethers in space since very high anode potentials are needed to draw a substantial ambient electron current across the magnetic field in the absence of collisions (or effective collisions due to turbulence). Isotropic quasineutral models of contactor clouds, extending over a region where the effective collision frequency ν_e exceeds the electron cyclotron frequency ω_{ce}, have low anode potentials but would collect very little ambient electron current, much less than the emitted ion current. A new model is presented for an anisotropic contactor cloud oriented along the magnetic field with $\nu_e < \omega_{ce}$. The electron motion along the magnetic field is nearly collisionless, forming double layers in that direction, whereas across the magnetic field the electrons diffuse collisionally and the potential profile is determined by quasineutrality. Using a simplified expression for ν_e due to ion acoustic turbulence, an analytic solution has been found for this model, which should be applicable to current collection in space. The anode potential is low, and the collected ambient electron current can be several times the emitted ion current.

Nomenclature

B_0 = ambient magnetic field
C = numerical factor relating electron thermal conductivity to electron transport
c_s = sound speed
E = electric field
e = charge on an electron
F_θ = azimuthal drag force on electrons
f_i = initial ionization fraction of source
g = focusing factor due to anisotropy
I = total current, $I_e + I_i$
I_e = electron current
I_i = ion current
J_e^∞ = ambient electron saturation current density
J_i = ion current density
$k_\perp$ = perpendicular wave number
L = length of tether
m_e = electron mass
m_i = ion mass
$n_{critical}$ = neutral density required for ignition
n_e = electron density
n_{ea} = ambient electron density
n_{ec} = contactor electron density
n_i = ion density
n_{ic} = contactor ion density
n_{source} = source plasma density
n_0 = neutral density
n_∞ = electron density at infinity
P_{load} = load power
R_{load} = load resistance
R_t = tether resistance

r = radial coordinate
r_{anode} = anode radius
r_{core} = radius at which electrons are collected for any model
r_{inner} = inner radius of double layer
r_{outer} = outer radius of double layer
r_{source} = source radius
r_1 = contactor cloud radius in anisotropic model
T_{ec} = contactor electron temperature
u_e = electron flow velocity
u_i = ion flow velocity
v_A = Alfven speed
v_d = electron azimuthal drift velocity
v_e = electron thermal velocity
v_r = radial velocity
v_z = axial velocity, parallel to the magnetic field
v_0 = orbital velocity
z = axial coordinate
z_0 = half length of anisotropic contactor cloud
β_e = ratio of electron pressure to magnetic pressure
γ = optical depth of source region to electron ionization
$\Delta\phi$ = potential drop across double layer
η = electrical efficiency of the tether
κ = cross-field electron thermal conductivity
λ_D = Debye length
$\lambda_{D,inner}$ = Debye length at r_{inner}
$\lambda_{D,outer}$ = Debye length at r_{outer}
μ = ratio ion mass to proton mass
ν_e = effective electron collision frequency
ξ = gain, I/I_i
ρ_e = electron gyroradius
σ = electron impact ionization cross section
ϕ = potential
ϕ_0 = anode potential
ϕ_{total} = total tether potential
ω_{ce} = electron cyclotron frequency
ω_{pe} = electron plasma frequency

I. Introduction

PLASMA contactors are plasma clouds that allow the passage of charge between an electrode and an ambient plasma. They have been proposed for use in power generating

devices such as electrodynamic tethers[1] because they may substantially reduce the impedance of the electron current collection from the ionosphere and make the emission of electrons much less energetically expensive than using an electron gun. In this paper we will concentrate on plasma contactors used at an anode to collect electrons in the ionosphere or some other ambient plasma. Such a contactor will emit ions as well as collect electrons. Two figures of merit for such a contactor are its impedance ϕ_0/I and the gain ξ, defined as

$$\xi = I/I_i(r_{anode})$$

The impedance determines the maximum power that can be generated by a tether since the total tether potential ϕ_{total} is fixed at $v_0 B_0 L$. If we ignore the ionospheric impedance and the impedance of electron emission, then

$$\phi_{total} = R_{load}I + R_t I + \phi_0(I)$$

The maximum power $R_{load}I^2$ at fixed ϕ_{total} and R_t is obtained when $R_{load} = R_t + d\phi_0/dI \approx R_t + \phi_0/I$. The power is greatest when the contactor impedance is lowest. The gain is important because it determines the rate at which gas must be used (to produce ions) for a given total current. If the gain is high, less gas is used to collect a given current.

Both the impedance and the gain will depend on the current. In general there is a tradeoff: at very low current, both high gain and low impedance are possible, but the power is low. Though at high current, high gain can be obtained only at the cost of very high impedance (again resulting in low power). Low impedance and high power are possible only with low gain. To illustrate these trends, we may consider the extreme limits. When the current is equal to the electron saturation current of the ambient plasma over the surface area of the physical anode, then the gain is infinite (since no ions need be emitted to draw this much electron current), and the contactor impedance is zero, but the power [for low Earth orbit (LEO) and practical tether and anode parameters] is at most tens of watts. Arbitrarily large current (and high power) may be obtained by emitting a large ion current, but unless the anode potential is high enough, it will not be possible to collect many electrons across the magnetic field, and the gain will approach unity. A basic goal of contactor research is to determine how large a gain is possible at a given power level. If it turns out that at the power levels of interest for tethers (typically tens of kW) the maximum gain is close to unity, then there is no point in using plasma contactors for current collection; in effect, the best plasma contactor is not better than an ion beam. If, on the other hand, gains at least a few times greater than unity are possible at power levels of interest, then plasma contactors are useful as current collectors for tethers. We will present theoretical results suggesting that this is the case; although the gains are only moderate, in the range of 2 to 10. These theoretical results pertain to a regime (collisionless electron motion along the magnetic field, collisional diffusion across the magnetic field), which we expect to be valid in LEO for high current contactors but for which there have been no ground-based experiments. Such experiments are very important for confirming the theory, or showing how it must be modified.

In previous work[2,3] it has been suggested that the plasma contactor cloud will consist of several different regions. First will be an inner core where the cloud will be isotropic because the two major directions of anisotropy, namely the Earth's magnetic field and the direction of motion of the source, will be shielded by the dense plasma from the contactor source. There will then be two outer regions where the two directions of anisotropy are manifested. Previously, it has generally been assumed that a substantial current of ambient electrons can be collected only from field lines that pass through the inner core region.[2,4] However, we will show in Sec. III that for conditions in LEO, it may be possible to collect a significant electron current from the outer core region, where the anisotropy due to the magnetic field is important.

There has been much debate about the size of the core region from which electrons can be collected. One estimate is obtained by matching the cloud density to the ambient density[5,6]

$$n_e(r_{core}) \approx n_{ea}$$

and another by taking magnetic field effects into account[7]

$$\nu_e(r_{core}) \approx \omega_{ce}$$

where ν_e is the radially dependent electron collision frequency (including effective "collisions" due to turbulence). A third estimate is obtained by requiring regularity of the self-consistent potential[8]

$$\left.\frac{\partial \phi}{\partial r}\right|_{r_{core}} \approx 0$$

and finally a fourth estimate comes by requiring a consistent space charge limited flow inside the core[9]

$$m_i n_i u_i^2 \mid_{r_{core}} \approx m_e n_e u_e^2 \mid_{r_{core}}$$

These diverse theories give a wide range of current enhancement factors for the plasma cloud.

If we assume a core cloud of radius r_{core}, then continuity of current gives

$$I = I_i(r_{anode}) + I_e(r_{anode}) = I_i(r_{core}) + I_e(r_{core})$$

and the gain is

$$\xi = \frac{I_e(r_{core})}{I_i(r_{anode})} + \frac{I_i(r_{core}) - I_i(r_{anode})}{I_i(r_{anode})} + 1$$

Plasma contactor clouds enhance or produce electron current flow through two possible paths. First (the first term on the right side of the equation), they can serve as virtual anodes through which electrons from far away can be drawn and collected to the real anode at the center of the cloud. Second, (the second term on the right side), the neutral gas associated with the cloud can become ionized creating electron-ion pairs. The electrons will be collected to the anode, and the ions will be repelled. For use in space with an electrodynamic tether, however, ionization of contactor neutrals is not an efficient use of neutral gas; if this is the only means by which the current is enhanced, then the same neutral gas can be used more efficiently by ionizing it internally in an ion source. Plasma contactors will be useful if they enable the ionosphere to supply electrons. The two sources of electrons in the ionosphere are the ionospheric plasma and the ionospheric neutrals. However, the mean free path for ionization of the ionospheric neutral gas is so long (many kilometers) that ionization of this gas on the length scale of the plasma contactor cloud is highly unlikely. For this reason we shall assume that all ionization associated with contactors is ionization of contactor neutral gas. Therefore plasma contactors can be useful with electrodynamic tethers only if they enhance current by collecting ambient electrons from the ionosphere. The collected electron current $I_e(r_{core})$ will generally be the saturation current times the area of the core cloud $4\pi r_{core}^2$, or if the contactor is only collecting electrons along magnetic field lines running into the core cloud, then $I_e(r_{core})$ will be the saturation current times $2\pi r_{core}^2$. (If, as we consider in Sec. III, the core cloud is not spherical but is elongated in the direction of the magnetic field, then r_{core} is the minor radius, across the magnetic field.) For this reason the size of r_{core} is crucial to the effectiveness of plasma contactors as electron collectors in space.

In Sec. II a collisionless, double-layer theory will be derived, along the lines of Wei and Wilbur[9] and Amemiya,[10] and it will be shown that this theory provides a good quantitative de-

scription of ground-based experiments at moderately low currents, but it will not be applicable to space-based contactors except at extremely low current and power. If the electrons are strictly collisionless, then the magnetic field prevents electrons from reaching the anode unless they originate on field lines that pass close to the anode (which limits the current that can be collected) or the anode potential is high enough to pull electrons across the magnetic field to the anode from some distance away. A necessary condition for this, which depends on the anode radius r_{anode}, was found by Parker and Murphy.[11] Another constraint on r_{anode} is that it must be less than the inner radius of the double layer. We will show that any spherically symmetric double layer with space-charge limited current greater than a very low limit (about 50 mA collected electron current corresponding to 1 mA emitted ion current for dayside equatorial LEO and even lower current for nightside) which satisfies these constraints must have an anode radius that is close to r_{core}. Such a plasma contactor would serve no purpose since it would hardly collect any more ambient electron current than the bare anode. This means that an unmagnetized, collisionless, space-charge limited, double-layer model, as analyzed by Wei and Wilbur,[9] cannot apply in space, except at very low currents, no matter how great the potential is. If the anode emits a current greater than this, at zero initial velocity (i.e., space-charge limited), and if the electrons are assumed to be collisionless, then the double layer cannot be spherically symmetric, regardless of the potential. Electron collection will be inhibited across the magnetic field, and the collected electron current will be lower than predicted by the Wei-Wilbur theory[9] for that anode potential and emitted ion current. Although a theory valid in this regime is not available, we can still obtain an upper limit on the collisionless electron current that can be collected and a lower limit on the anode potential for a given ion current by assuming that the Parker-Murphy condition is marginally satisfied for a double layer obeying the equations of Wei and Wilbur and ignoring the constraint that the inner radius of such a double layer must occur at a greater radius than r_{anode}. We then obtain an upper limit to the power that can be generated by a plasma contactor collecting electrons to a 20-km-long tether in space in the absence of electron collisions. This maximum power is quite low, only a few hundred watts, less than an order of magnitude above the power that can be generated by a tether without a plasma contactor using a bare anode to collect electrons.

At higher emitted ion current, there will be a region where the electrons cannot go straight to the anode but where ambient electrons will be trapped to keep the plasma quasineutral. These electrons will remain trapped for a time long compared to the time it would take for an unmagnetized electron to go straight to the anode. If there are effective collisions due to instabilities, some of these trapped electrons may be able to diffuse to the anode, and the collected electron current may be much greater than what would be found in the collisionless model.

In Sec. III, we will describe work on a model of the outer core region in which the motion along the magnetic field is collisionless, forming a double layer, but the motion across the magnetic field is collisional and quasineutral. This model, which is expected to be applicable to contactors in space, suggests that significant current may be collected from this outer core region with low contactor impedance. Unfortunately there are, to our knowledge, no experiments in this regime, to which the theory can be compared. Conclusions will be presented in Sec. IV.

II. Double-Layer Theory and Implications

A. Collisionless Unmagnetized Model

Ground-based experiments in which double layers are seen are well described by a collisionless unmagenetized model as we will show. A schematic radial potential profile for such a model is shown in Fig. 1. We assumed that the potential is
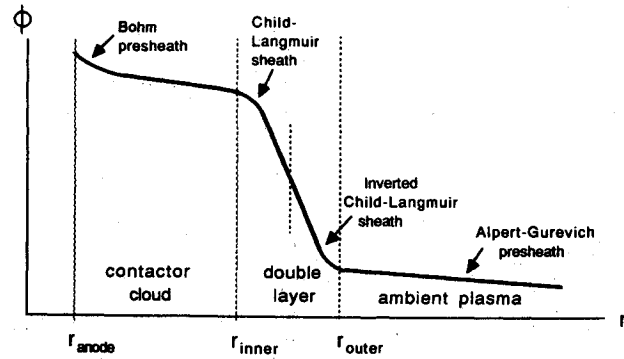

Fig. 1 Schematic radial potential profile for collisionless unmagnetized double layer; the Bohm presheath is described in Refs. 12 and 13, and the Alpert-Gurevich presheath in Ref. 14.

monotonic; therefore there are two components of plasma, an ambient component and a contactor component. The ambient ions and electrons are Maxwellian at positions r well beyond the double layer with ion and electron temperatures T_{ia} and T_{ea} and density n_∞. The contactor plasma has Maxwellian electrons at temperature T_{ec} and cold ions streaming radially out from a plasma source localized near the anode with ion current I_i. The potential drop ϕ_0 between the source at $r = r_{source}$ and the ambient plasma at $r \to \infty$ is assumed to be much greater than any of the temperatures, and the radius at which the double-layer forms is assumed to be much greater than a Debye length. With these assumptions, the plasma is quasineutral everywhere except inside the double layer at $r_{inner} < r < r_{outer}$. (Here r_{outer} is the radius, called r_{core} in the Introduction, at which the ambient electron saturation current is collected.) Inside the contactor cloud at $r < r_{inner}$, there are no ambient ions, and the density of ambient electrons, which have been accelerated in the double layer, is much less than the density of contactor electrons; therefore, quasineutrality requires $n_{ec}(r) = n_{ic}(r)$. The densities of contactor electrons and ions are related to the potential ϕ (defined relative to $r \to \infty$) by

$$n_{ec} = n_{source}\exp[(\phi - \phi_0)/T_{ec}] \tag{1}$$

$$n_{ic} = n_{source}(r_{source}/r)^2[1 + (\phi_0 - \phi)/T_{ec}]^{-1/2} \tag{2}$$

where we have assumed that ions are emerging from the source at the sound speed $(T_{ec}/m_i)^{1/2}$, due to acceleration in a Bohm presheath, and we have neglected any ionization or recombination occurring at $r > r_{source}$. Setting the right sides of Eqs. (1) and (2) equal to each other gives a transcendental equation for $\phi(r)$. It is evident that for $r \gg r_{source}$,

$$\phi(r) \approx \phi_0 - 2T_{ec}\ln(r/r_{source}) \tag{3}$$

so the potential only drops a few times T_{ec} inside the contactor cloud, much less than the total potential drop. The source density n_{source} is related to the ion current I_i by

$$I_i = 4\pi r_{source}^2 e n_{source}(T_{ec}/m_i)^{1/2} \tag{4}$$

Outside the double layer, at $r > r_{outer}$, the ambient electron density decreases from n_∞ as r decreases because no electrons are emitted from the double layer. We assume that there are no sources of electrons or collisions which could fill in the resulting empty region of velocity space. From quasineutrality, the ambient ion density must also decrease as r decreases (even if the density of contactor ions, accelerated in the double layer, is small compared to the ambient ion density); so, the potential must rise by an amount on the order of T_{ia}. If T_{ia} is much less than T_{ea}, then the ambient electron density is

not affected by the potential; therefore it is reduced from n_∞ by a simple geometric factor

$$n_{ea}(r) = \tfrac{1}{2} \, n_\infty [1 + (1 - r_{outer}^2/r^2)^{1/2}] \qquad (5)$$

and the potential is given by

$$\phi(r) = T_{ia} \ln(n_\infty/n_{ea}) \qquad (6)$$

(This rise in potential going from infinity to r_{outer} causes the ambient electrons to become supersonic by the time they reach r_{outer} so that they satisfy the Bohm sheath condition.[12,13] This potential was calculated by Alpert et al.[14] for the case $T_{ia} = T_{ea}$ so we have labeled this region the "Alpert-Gurevich presheath" in Fig. 1.) The potential drop from r_{outer} to ∞ is just $T_{ia} \ln 2$, much less than the total potential drop. Most of the potential drop must therefore occur in the double layer. Within the double layer $r_{inner} < r < r_{outer}$, the plasma is not quasineutral, and Poisson's equation (for spherical symmetry)

$$\frac{1}{r^2} \frac{d}{dr} r^2 \frac{d\phi}{dr} = 4\pi(n_e - n_i) \qquad (7)$$

must be satisfied subject to the boundary conditions that ϕ and $d\phi/dr$ be continuous at r_{inner} and r_{outer}. These four boundary conditions specify a solution to the second-order differential equation, and the values of the free parameters r_{inner} and r_{outer}. Since most of the drop in potential occurs in the double layer, to a good approximation the boundary conditions are

$$\phi(r_{inner}) = \phi_0 - 2T_{ec} \ln(r_{inner}/r_{source}) \qquad (8)$$

$$\phi(r_{outer}) = 0 \qquad (9)$$

$$\frac{d\phi}{dr} = 0 \text{ at } r_{inner} \text{ and } r_{outer} \qquad (10)$$

If, as we have assumed, $T_{ia} \ll T_{ea}$, then the ambient ion density drops much more quickly than the ambient electron density as the potential starts to rise going inward from r_{outer}, and we can neglect the ambient ion density in Eq. (7). Similarly, since the energy of the contactor ions is greater than T_{ec} at r_{inner}, even if only by a logarithmic factor, the contactor electron density drops much more quickly than the contactor ion density in going outward from r_{inner}, and to rough approximation we can neglect the contactor electron density in the double layer. In the double layer, then, we must solve Poisson's equation, Eq. (7), with

$$n_e = \frac{n_\infty}{2} \frac{r_{outer}^2}{r^2} \exp\left(\frac{\phi}{T_{ea}}\right)\left[1 - \text{erf}\left(\sqrt{\frac{\phi}{T_{ea}}}\right)\right] \qquad (11)$$

$$n_i = n_{source} \frac{r_{source}^2}{r^2} \left[\frac{\phi_0 - \phi(r)}{T_{ec}}\right]^{-1/2} \qquad (12)$$

An approximate analytic solution, which provides some physical insight, may be found when the double layer is thin, i.e., $r_{outer} - r_{inner} \ll r_{inner}$. Then, in the vicinity of r_{inner}, for $\lambda_D \ll r - r_{inner} \ll r_{outer} - r_{inner}$, the potential approximates a Child-Langmuir sheath with negligible n_e

$$\phi(r_{inner}) - \phi(r) \approx 3^{4/3} T_{ec} \ln\left(\frac{r_{inner}}{r_{source}}\right)\left(\frac{r - r_{inner}}{\lambda_{D,inner}}\right)^{4/3} \qquad (13)$$

where

$$\lambda_{D,inner}^2 = \frac{T_{ec} \ln(r_{inner}/r_{source})}{2\pi e^2 n_{source}}\left(\frac{r_{inner}}{r_{source}}\right)^2 \qquad (14)$$

is the Debye length at r_{inner}. In the vicinity of r_{outer}, for $\lambda_D \ll r_{outer} \; r - \ll r_{outer} - r_{inner}$, the potential approximates an

inverted Child-Langmuir sheath with negligible n_i

$$\phi(r) \approx \frac{3^{4/3}}{2} T_{ea} \left(\frac{r_{outer} - r}{\lambda_{D,outer}}\right)^{4/3} \qquad (15)$$

where

$$\lambda_{D,outer}^2 = \frac{T_{ea}}{2\pi e^2 n_\infty} \qquad (16)$$

is the Debye length at r_{outer}. The transition from Eq. (13) to Eq. (15) occurs when $n_e \approx n_i$, which is to say at the point where the two expressions for $\phi(r)$, Eqs. (13) and (15), have second derivatives that are equal in magnitude (but with opposite signs). At this point, the two expressions for $\phi(r)$ must have the same first derivative. This means that the transition from Eq. (13) to Eq. (15) must occur halfway between r_{inner} and r_{outer} with $\phi(r)$ antisymmetric about this point, and the coefficients in front of the two expressions for $\phi(r)$ must be equal

$$2T_{ec} \ln(r_{inner}/r_{source})\lambda_{D,inner}^{-4/3} = T_{ea}\lambda_{D,outer}^{-4/3} \qquad (17)$$

Equation (17) leads immediately to the well known double-layer requirement

$$I_e/I_i = (m_i/m_e)^{1/2} \qquad (18)$$

where $I_e = 2\pi r_{outer}^2 J_e^\infty$, and $J_e^\infty = en_\infty(2\pi T_{ea}/m_e)^{1/2}$ is the ambient electron saturation current density. In other words, the contactor cloud will expand freely until the ion current density $I_i/4\pi r^2$ is equal to the ambient electron saturation current times $(m_e/m_i)^{1/2}$. If $T_{ea} \approx T_{ec}$, then this will occur when the density of the contactor plasma is comparable to the density of the ambient plasma.

From Eqs. (13), (15), and (17), the width of the double layer is related to the potential drop $\Delta\phi = \phi(r_{inner}) - \phi(r_{outer})$ by

$$r_{outer} - r_{inner} = \frac{2}{3} \lambda_{D,outer}\left(\frac{\Delta\phi}{T_{ea}}\right)^{3/4} \qquad (19)$$

and these results are valid only if the width given by Eq. (19) is much less than r_{inner}. If this condition is not satisfied, then Poisson's equation must be solved numerically, as had been done by Wei and Wilbur[9] and by Williams,[15] and in this case I_e/I_i will be smaller than $(m_i/m_e)^{1/2}$.

B. Comparison with Experiment

The model outlined above is in good agreement with the ground-based experiments of Wilbur[16] in those conditions where double layers were seen. In these experiments, the anode had a radius $r_{anode} = 6$ cm, but the effective source radius, where most of the ionization occurred, was $r_{source} \approx 2$ cm. The anode potential ϕ_0 could vary from 0 to 70 V, and the collected electron current could vary from 0 to 1 A. (At higher current, the effective collision frequency, due to streaming instabilities, was too high for collisionless, double-layer theory to be valid.) Neutral gas, xenon, was introduced at the center of the anode at a rate that could vary from 1.8 to 13.7 sccm (standard cubic centimeters per minute), which corresponded to a neutral density ranging from 3×10^{11} to 10^{12} cm^{-3} concentrated within r_{source} of the origin. For ϕ_0 above some critical value, which depended on the neutral density, ambient electrons accelerated in the double layer had enough energy to ionize the gas, and the contactor cloud underwent a transition to an "ignited mode" in which this ionization was the major source of emitted ion current. The electron temperature, density, and plasma potential were measured as functions of position. The ambient ion temperature was much lower than the electron temperatures.

In a typical case, with $\phi_0 = 37$ V, most of the potential drop, 25 V, occurred in a double layer (more or less spherical) located between $r_{inner} = 8$ cm and $r_{outer} = 11$ cm. The rest of the potential drop occurred between the anode and r_{inner}. The potential profile was virtually flat outside r_{outer}. The ambient electron temperature was 5.5 eV, and the ambient electron density was 3×10^7 cm^{-3}. These electrons have a Larmor radius of about 15 cm in the Earth's magnetic field, which is greater than $r_{outer} - r_{inner}$, and once they cross the double layer, they have a Larmor radius of about 50 cm, which is greater than $r_{outer}^2/2r_{anode}$; therefore the electrons can easily reach the anode according to the Parker-Murphy criterion,[11] and the assumption of our model of unmagnetized electrons is valid. The assumption of collisionless electrons was also marginally satisfied if we estimate the effective collision frequency to be $\nu_e \approx 10^{-2}\omega_{pe}$. At r_{outer} we find $\nu_e = 3 \times 10^6$ s^{-1}, and the electron mean free path is about 30 cm, greater than the width of the double layer, whereas at r_{inner} we find $\nu_e = 2 \times 10^6$ s^{-1}, and the mean free path of the accelerated ambient electrons is about 1 m greater than r_{inner}. If the effective collision frequency is less than that taken here, then the assumption of collisionless electrons is easily satisfied. Note that at densities a few times higher, the electron mean free path would be comparable to the double-layer width, and double layers could not exist. This is in agreement with observations at currents above 1 A. There was also a 40-eV ambient electron component (the "primary" electrons) of density 3×10^6 cm^{-3}. Such a component of electrons was not included in our model, but their effect can be included by using an effective $T_{ea} \approx 9$ eV, which would give the same electron saturation current as that obtained from the 5.5 eV and 40 eV components.

The collected electrons current 370 mA was in good agreement with this electron saturation current integrated over the area of the double layer $2\pi r_{outer}^2$ (not $4\pi r_{outer}^2$ since it was a half sphere). The electrons in the contactor cloud had a temperature $T_{ec} = 2$ eV and a density which went from 8×10^8 cm^{-3} at r_{source} down to 2×10^7 cm^{-3} at r_{inner}. This ratio of $n_e(r_{source})/n_e(r_{inner})$ is close to $(r_{inner}/r_{source})^2\{[\phi_0 - \phi(r_{inner})]/T_{ec}\}^{1/2}$, the value given by Eq. (2). The emitted ion current I_i would then be $2\pi r_{source}^2 e n_e(r_{source})(T_{ec}/m_i)^{1/2} = 0.4$ mA, fairly close to the ion current required by Eq. (17), $(m_e/m_i)^{1/2}I_e = 0.7$ mA. The observed width of the double layer $r_{outer} - r_{inner} \approx 3$ cm is a few times greater than the width of 0.6 cm predicted by Eq. (18), but it is likely that the measured width is smeared out by fluctuations in the position of the double layer. Such fluctuations could be because of some intrinsic property of the double layer that would cause it to oscillate around equilibrium[17] instead of asymptotically approaching equilibrium. Such behavior is likely to be associated with nonmonotonic potentials,[18] a feature that we have not included in our model. The fluctuations could also be caused by a more mundane effect, such as fluctuations in the gas feed. It would be of interest to try to measure such fluctuations and to determine their cause.

C. Limitations of Wei and Wilbur Model Due to Magnetized Electrons

In Wilbur's ground-based experiments,[16] the Larmor radius of the ambient electrons in the Earth's 0.3 G magnetic field is about 20 cm, much greater than the 3-cm thickness of the double layer, so the magnetic field will not significantly deflect the electrons as they cross the double layer. Once they cross the double layer, they will have a Larmor radius of about 50 cm, and in the 8 cm they have to traverse to get to the anode, they will be deflected by about $\frac{1}{2}(8)^2/50 = 0.7$ cm, less than the 6-cm radius of the anode; consequently the magnetic field will not inhibit the electrons from getting to the anode.[11] Hence our model, which assumed unmagnetized electrons, ought to be valid. An additional requirement of our model $r_{inner} > r_{anode}$ is also satisfied in Wilbur's experiments.

In space, on the other hand, the ambient electron temperature, at least in the equatorial region, is much less, only about 0.1 eV, so that Larmor radius is about 2.5 cm, and the density is much less than in the ground-based experiments (about 10^5 cm^{-3} rather than 3×10^7 cm^{-3}). Therefore, collecting an electron current of several amps from the ambient plasma will require r_{outer} of tens of meters, much greater than the electron Larmor radius. The electrons can traverse such a distance only if they undergo collisions (or effective collisions due to some kind of instability) or if they can gain enough energy as they cross the double layer to remain, in effect, unmagnetized until they reach the anode. We have considered the latter possibility and have found that, even with rather optimistic assumptions, it requires a sheath impedance that is undesirably large since it would result in most of the tether potential drop occurring in the sheath. We conclude that effective collisions of some kind are needed in a plasma contactor in space in order to collect a large electron current from the ambient plasma at a reasonable impedance.

Parker and Murphy[11] have shown that in the absence of collisions, and for $e\phi_0 \gg T_{ea}$, a necessary condition that must be satisfied for electrons at r_{outer} to reach the anode is

$$r_{outer}^2/r_{anode}^2 < 1 + (8e\phi_0/m_e\omega_{ce}^2 r_{anode}^2)^{1/2} \qquad (20)$$

Equation (20) is also a sufficient condition if all of the potential crop occurs in a thin double layer at r_{outer}. If the double layer is thick, or if a significant part of the potential drop occurs in the quasineutral regions on either side of the boundary layer, then an even more stringent condition must be satisfied in order for electrons to reach the anode. Another condition that must be satisfied is $r_{inner} \geq r_{anode}$. It turns out that for most parameters of interest, this condition and Eq. (20) cannot both be satisfied for a spherically symmetric, space-charge limited, collisionless double layer, as described by Wei and Wilbur[9] and Amemiya.[10] This is true except at very low currents or for anodes with r_{anode} almost equal to r_{outer}. If higher ion currents are emitted from an anode (with $r_{anode} \ll r_{outer}$) with zero initial velocity, and there are no collisions or turbulence allowing electrons to be transported across the magnetic field, then a spherically symmetric double layer cannot develop, no matter how great the potential is. Electron collection will necessarily be inhibited in the direction across the magnetic field; in this direction the potential profile will not follow the form found by Wei and Wilbur[9] because the collected ambient electron current will not be space-charged limited but will be limited by magnetic field effects. A theory giving the electron current and potential in this anisotropic collisionless regime is not available. However, if we ignore the requirement that $r_{inner} > r_{anode}$ and assume that only Eq. (20) and the Wei-Wilbur equations must be satisfied, then we can obtain an upper limit for the electron current that can be collected and a lower limit for the potential for a given ion current and anode radius.

The electron current I_e is related to r_{outer} by

$$I_e = 2\pi r_{outer}^2 J_e^\infty \qquad (21)$$

where $J_e^\infty = en_{ea}(T_{ea}/2\pi m_e)^{1/2}$ is the ambient electron saturation current. We have calculated what the impedance of the double layer will be assuming Eq. (20) is barely satisfied for $r_{anode} = 10$ cm. If, as turns out to be true, the resulting impedance is too high to make an efficient plasma contactor, we will know that we should look at plasma contactors in which the electrons undergo collisions (or are subject to turbulence which causes effective collisions) and diffuse into the anode rather than going into the anode directly.

Using Eq. (21) for I_e, assuming Eq. (20) is barely satisfied, and using Wei and Wilbur's calculation,[9] which relates r_{outer}/r_{inner} uniquely to I_e/I_i, we can find ϕ_0 and I_e for a given I_i and electron saturation current J_e^∞. Since J_e^∞ depends only on the properties of the ionosphere in low Earth orbit, both I_e and ϕ_0 are determined by I_i. These values really represent an upper limit for I_e and lower limit for ϕ_0 since Eq. (20) is only a
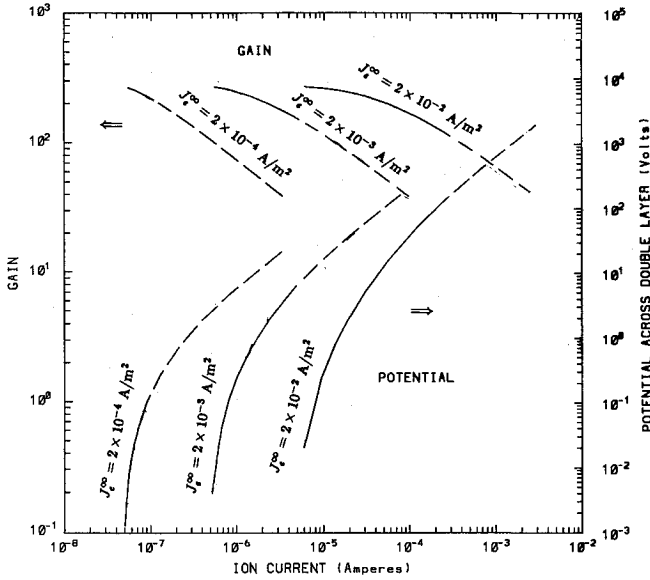
Fig. 2 Gain and lower limit on potential drop as functions of the emitted argon ion current and the electron saturation current for collisionless double layer with space charge limited current, marginally satisfying the Parker-Murphy condition with a 10-cm anode radius using Eqs. (20) and (21) and Fig. 5 of Ref. 9.
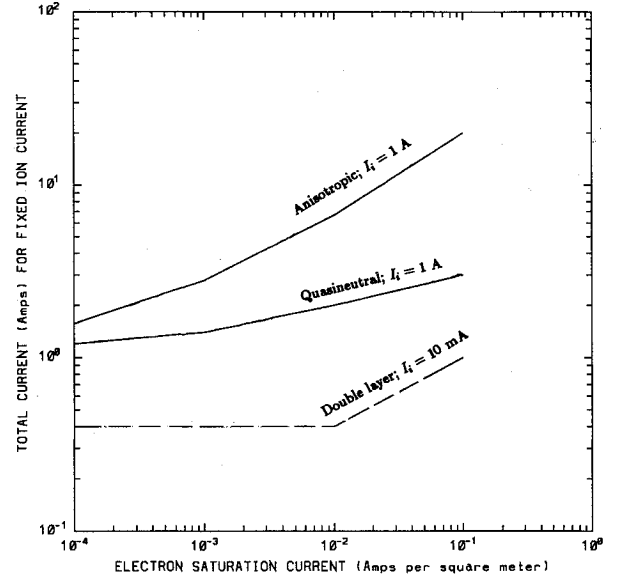

Fig. 3 Total collected current vs electron saturation current with the emitted ion current held constant for the collisionless double-layer model (upper limit), the isotropic quasineutral model,[4] and the anisotropic contactor model.

necessary condition, not a sufficient condition, for collisionless electrons to reach the anode and since we ignored the requirement that $r_{inner} > r_{anode}$. The gain and potential drop are obtained by imposing the Parker-Murphy requirement and the limited source requirement [Eq. (21)] on the Wei and Wilbur results.

In Fig. 2 we show the gain ξ against the ion current for argon and for a range of electron saturation current densities which span the range experienced in an equatorial LEO. The gain is somewhat less than $(m_i/m_e)^{1/2} = 272$ for argon and is weakly dependent on the ion current. We also show the associated potential drop through the double layer, which is really a lower limit on the potential drop. Typical potential drops are in the range of thousands of volts for ion currents in the milliampere range. In Figs. 2–4, the curves are dashed in the regime where Eq. (20) cannot be satisfied for a collisionless double layer with space-charge limited current except by violating $r_{inner} \geq r_{anode}$. Note that the curves are dashed except at the smallest ion currents, showing that a collisionless unmagnetized double layer with space-charge limited current is not possible for most parameters of interest in the LEO. This conclusion does not depend on r_{anode}. Making $r_{anode} < 10$ cm would only make things worse since, for a fixed ion current, r_{inner} would shrink faster than r_{anode}. Making r_{anode} much greater than 10 cm would allow higher ion and electron currents and satisfy Eq. (20) and $r_{inner} > r_{anode}$. However, for $J_e^\infty \leq 2 \times 10^{-2}$ A/m², this could only be done if r_{anode} were nearly equal to r_{outer} in which case the plasma contactor would serve no purpose. Another way to show that this conclusion does not depend on r_{anode} is to use Eqs. (19) and (20) with $\phi_0 \approx \Delta\phi$, $r_{anode} = r_{inner}$, and $r_{outer} \approx 2r_{inner}$. Combining these equations gives us

$$\phi_0 \approx \frac{T_{ea}}{e}\left(\frac{\omega_{pe}}{\omega_{ce}}\right)^4 \qquad (22)$$

$$r_{outer} \approx \left(\frac{\omega_{pe}}{\omega_{ce}}\right)^2 \rho_e \qquad (23)$$

where ω_{pe} is the ambient electron plasma frequency and ρ_e is the ambient electron gyroradius. Equations (18) and (21) then give

$$I_i < 2\pi\left(\frac{m_e}{m_i}\right)^{1/2} J_e^\infty \left(\frac{\omega_{pe}}{\omega_{ce}}\right)^4 \rho_e^2 \qquad (24)$$

as the maximum ion current for which a collisionless unmagnetized double layer with space-charge limited current is possible. This ion current depends only on ambient quantities and m_e/m_i, not on r_{anode} or ϕ_0, and is never greater than about 1 mA for LEO.

In Fig. 3 the total current is shown as a function of the electron saturation current density. The curve obtained for the collisionless double layer (really an upper limit) is shown for a fixed ion current of 10 mA. For comparison, we also show the total current for the isotropic quasineutral model described in Ref. 4, and for the anisotropic contactor mode described in Sec. III, for a fixed ion current of 1 A. This figure compares the realistic range of operation for the three models in typical ambient electron saturation current densities. A significant feature of this figure is that as the source varies by two orders of magnitude from 2×10^{-4} A/m² to 2×10^{-2} A/m², the total current (which is almost all collected electron current) varies by only a factor of 1.5 for the collisionless double-layer model. This would seem to invalidate one of the conclusions in Ref. 1, which was that plasma contactors would not be useful on the nightside of an equatorial LEO because the collected current would drop to almost nothing. Here the double layer moves out as the electron pressure drops so that the collected electron current is almost the same. On the other hand, if we took into account the actual requirements for electrons to reach the anode, rather than only using the Parker-Murphy condition, then it is likely that at low saturation current the double layer would be inhibited from moving out so far, and the collected electron current would be more sensitive to saturation current. Except for the upper end of the range of saturation current, the actual electron current that could be collected without collisions is certainly far less than the upper limit shown in Fig. 3. For the anisotropic collisional contactor model, which is more relevant for high current plasma contactors in LEO, Fig. 3 shows that the total current is about 4 times higher, and the collected electron current is about 10 times higher on the dayside ($J_e^\infty \approx 2 \times 10^{-2}$ A/m²) than on the nightside ($J_e^\infty \approx 2 \times 10^{-4}$ A/m²).

In Fig. 4 the current voltage characteristic is shown for the range of electron saturation current densities. At constant current in the milliampere range, the voltage is seen to vary by two or three orders of magnitude for one order of magnitude variation in electron saturation current for the collisionless double layer. At constant voltage, the current is roughly linear with the electron saturation current. Ampere range currents
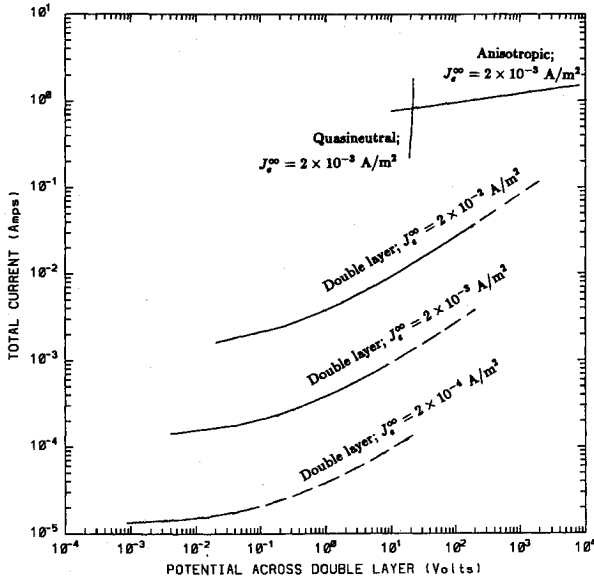
Fig. 4 Total current vs potential drop for the collisionless double-layer model, the isotropic quasineutral model,[4] and the anisotropic contactor model.

(which are mainly electrons) require tens of thousands of volts of potential drop, even for the highest value of the electron saturation current. These curves represent an upper limit on the electron current for a given potential or a lower limit on the potential for a given electron current. For currents greater than about 50 mA, the space charge limited collisionless double-layer model on which these curves are based cannot satisfy both Eq. (20) and $r_{inner} > r_{anode}$; the actual potential needed to collect such currents, in the absence of collisions, would be far greater than the lower limits shown in Fig. 4. Curves for the isotropic quasineutral model and anisotropic model discussed in Ref. 4 and Sec. III are shown for comparison.

With the use of these results we can calculate an upper limit for the current that could flow through a tether using a plasma contactor to collect electrons. A circuit diagram for a tether is shown in Fig. 1 of Ref. 19. The total potential drop ϕ_{total} across the contactor, tether, load, and electron gun (or electron emitting contactor) is fixed by the length L of the tether, the Earth's magnetic field $B_0 = 0.33 \times 10^{-4}T$, and the orbital velocity of the spacecraft $v_0 = 8$ km/s. For $L = 20$ km, we find $\phi_{total} = v_0B_0L = 5333$ V. The potential across the load is $\phi_{load} = R_{load}(I_i + I_e)$. The potential across the tether is $R_t(I_i + I_e)$ where we take the tether impedance $R_t = 200 \, \Omega$. We could include the radiation impedance,[20] but this is typically only about $10 \, \Omega$ so may be neglected compared to the tether impedance. We also neglect the impedance of the electron gun or electron emitting contactor. If we assume a typical dayside ionosphere with $J_e^\infty = 2 \times 10^{-2}$ A/m^2, a good fit to the numerical results in Fig. 4 is $\phi_0 = b(I_i + I_e)^{2.08}$ where $b = 1.8 \times 10^5$. For a given load R_{load}, the current $I = I_i + I_e$ may be found by solving

$$\phi_{total} = R_{load} I + R_t I + bI^{2.08} \qquad (25)$$

and we may then find the power across the load $P_{load} = R_{load}I^2$ and the efficiency $\eta = R_{load}I/\phi_{total}$ as functions of R_{load}. (This definition of efficiency neglects the energy needed to produce the ions, but that is justified since this energy, about 50 eV, is much less than the potential drop across the double layer unless $\eta \approx 99\%$.) Table 1 shows P_{load} and ξ as functions of the efficiency $\eta = R_{load}I/\phi_{total}$.

The maximum power to the load is 400 W, but this occurs when the efficiency is only 70%. As noted in Ref. 1, in order for tethers to be competitive with other power systems in space, it is necessary for them to operate at high efficiency, at

least 80% or 90%. This is because all of the power has to be made up by periodically boosting the tether but only the load power can be used. If we desire an efficiency of 85%, then the maximum load power we can obtain is only 320 W. The maximum power will in fact be much less than this since Eq. (20) is not a sufficient condition for electrons to get across the magnetic field to the anode,[11] and is known to be far from sufficient in the regime where $r_{outer} \gg r_{inner}$, which is true at the maximum power. Also, the requirement that $r_{inner} > r_{anode}$ is far from satisfied at the ion current needed for maximum power.

We conclude that it is not possible to design a high-power contactor which draws electrons straight across a double layer without collisions. Instead we should consider designs where collisions (or, more realistically, effective collisions due to instabilities of some kind) transport electrons across the magnetic field to the anode.

D. Conditions for Ignited Plasma

The calculations so far with the double-layer model have all been for a totally ionized plasma. For a partially ionized plasma, it is possible to include the effect of ionization and to show when the plasma will ignite. If we assume the neutral gas is expanding radially from the source at r_{source}, and that only a small fraction of it gets ionized, then the neutral density varies with radius as $n_0(r) = n_0(r_{source})(r_{source}/r)^2$. We apply conservation of mass from r_{source} to r_{inner} to obtain

$$I_e(r) = I_e(r_{inner}) \exp\left[\gamma(\Delta\phi)\left(\frac{r_{source}}{r} - \frac{r_{source}}{r_{inner}} \right)\right] \qquad (26)$$

where $\gamma(\Delta\phi) = n_0(r_{source})r_{source}\sigma$. Here the electron ionization cross section σ is to be evaluated at a typical energy for an incoming ambient electron $\Delta\phi + T_{ea}$. From conservation of current we obtain the gain as

$$\xi = 1 + \frac{[\xi(r_{inner}) - 1]\exp\left[\gamma\left(1 - \frac{r_{source}}{r_{inner}}\right)\right]}{1 + [\xi(r_{inner}) - 1]\left\{1 - \exp\left[\gamma\left(1 - \frac{r_{source}}{r_{inner}}\right)\right]\right\}} \qquad (27)$$

where $\xi(r_{inner}) = I/I_i(r_{inner})$. The ion current at the source in terms of ion current just inside the double layer is

$$\frac{I_i(r_{source})}{I_i(r_{inner})} = 1 + [\xi(r_{inner}) - 1]\left\{1 - \exp\left[\gamma\left(1 - \frac{r_{source}}{r_{inner}}\right)\right]\right\} \qquad (28)$$

In order to interpret the calculations in Fig. 2 with ionization present, we must interpret the ion current in the abscissa as $I_i(r_{inner})$ and the gain as $\xi(r_{inner})$. The relationship in terms of the ion current emitted at the source is given above. It is apparent that there may be no positive solution of the source ion current for a given ion current at the double layer. Physically this will occur when there is so much neutral gas that the mixed gas-plasma flow ignites giving an avalanche of ion current. The ion current and collected electron current will continue to increase and cannot reach a steady state until the collisionless double-layer model is no longer valid. By setting the source ion current to zero, we can obtain this critical

Table 1 Load power against efficiency of double-layer contactor

η	I_i(mA)	ξ	I(A)	P_{load}(W)
0.1	7	26	0.18	100
0.3	6	27	0.16	260
0.5	5	35	0.14	380
0.7	2.5	44	0.11	400
0.9	0.8	75	0.06	290

neutral density for ignition as

$$n_{\text{critical}} = \frac{-\ell n[1 - 1/\xi(r_{\text{inner}})]}{(1 - r_{\text{source}}/r_{\text{inner}})} \frac{1}{r_{\text{source}}\sigma} \qquad (29)$$

If we relate the source neutral density to the ion flow rate and initial fractional ionization (f_i), we obtain ignition for

$$I_i(r_{\text{inner}}) > \frac{4\pi r_{\text{source}}^2 c_s e f_i}{1 - f_i} n_{\text{critical}} \qquad (30)$$

Taking $r_{\text{source}} = 0.1$ m, $c_s = 4.89 \times 10^3$ m/s, $\sigma = \sigma_{\text{max}} = 3.21 \times 10^{-20}$ m^{-2} (for ionization of argon) and $f_i = 10^{-4}$, which is typical of hollow cathode devices, we find that the critical ion current is much greater for a given gain than the ion currents for which the collisionless double-layer model is valid in LEO, shown as solid curves occur in Fig. 2. Hence, ignition will never occur in this regime. Ignition might be possible in the regime of higher ion current and lower gain typical of the anisotropic collisional contactor model described in Sec. III.

III. Anisotropic Contactor Model

Hastings and Blandino[4] considered a model where electrons were transported across the magnetic field by effective collisions due to instabilities and assumed that such transport could occur only out to a distance r_{core} where the effective collision frequency ν_e was greater than the electron cyclotron frequency ω_{ce}. With that model, they found that the collected ambient electron current for typical parameters in LEO was less than the emitted ion current. Here, we consider the possibility that electrons can be collected from a more distant region where $\nu_e < \omega_{ce}$. In that region the contactor cloud will be anisotropic extending further in a direction along the magnetic field than across the magnetic field. We therefore use cylindrical coordinates z and r, where r now refers only to the distance across the magnetic field, not to the total distance from the anode as it did in previous sections. We assume that the plasma density in the cloud is still great enough to short out the electric field due to the orbital velocity so that the cloud will be cylindrically symmetric. (At larger distances from the anode, the effects of the orbital motion induced electric field will become important, and the cylindrical symmetry will be broken.) In this region the electron velocity will be mostly azimuthal at the drift velocity

$$v_d = \frac{e}{m_e\omega_{ce}} \frac{\partial \phi}{\partial r} - \frac{1}{m_e\omega_{ce}} \frac{\partial T_e}{\partial r} - \frac{T_e}{m_e\omega_{ce}n_e} \frac{\partial n_e}{\partial r} \qquad (31)$$

For parameters of interest, this drift velocity is much greater than the radial flow velocity of the emitted ions, which are effectively unmagnetized since we assume that the scale lengths are all much less than an ion Larmor radius. The velocity difference between the electrons and ions will then be nearly in the azimuthal direction. This relative cross-field drift velocity of magnetized electrons and unmagnetized ions can give rise to several instabilities, among them the ion acoustic instability (both $k_\perp \rho_e > 1$ and $k_\perp \rho_e < 1$ varieties), the Buneman instability, the electron cyclotron drift instability (also known as the beam cyclotron instability), the modified two-stream instability, and the lower hybrid drift instability. Which of these instabilities dominates depends on such parameters as T_e/T_i, v_d/c_s, v_d/v_e, β_e, ω_{pe}/ω_{ce}, and v_d/v_A. These instabilities will give rise to turbulent azimuthal electric fields, which will exert an azimuthal drag force $F_\theta = \nu_e m_e v_d$ on the electrons giving rise to a drift in the $F \times B_0$ (inward radial) direction at velocity

$$v_r = \frac{\nu_e}{\omega_{ce}} v_d \qquad (32)$$

We will assume that the potential drop in the plasma cloud is very much greater than the ion temperature T_i, which is typically only a few electron volts. Since, as we will show later, T_e tends to be only a few times less than ϕ_0, this implies that $T_e/T_i \gg 1$, except perhaps near the edge of the cloud. Also $c_s \ll v_d \ll v_e$. In these circumstances, we expect the $k_\perp \rho_e > 1$ ion acoustic instability to dominate (this is the same as the ion acoustic instability in an unmagnetized plasma). The effective collision frequency ν_e for this instability in its nonlinear saturated state scales with density like ω_{pe} and is independent of c_s/v_d for $c_s \ll v_d$, but there is some uncertainty as to its dependence on T_e/T_i and v_d/v_e. We will simply assume that

$$\nu_e \approx 10^{-2}\omega_{pe} \qquad (33)$$

independent of the other parameters. There is considerable theoretical and experimental evidence that the effective collision frequency due to ion acoustic turbulence is proportional to ω_{pe} and somewhat weaker evidence that the constant of proportionality should be 10^{-2} as in Eq. (33).

This evidence is discussed in Ref. 7. In addition, we note that particle simulations of saturated ion acoustic turbulence in infinite medium[21-23] generally give effective collision frequencies of this magnitude and that experimental observations of collisionless shocks are in agreement with this result.[24] In a plasma contactor, the scale lengths are not infinite compared to the wavelengths of the unstable modes, the geometry differs from that of Ref. 24, and Eq. (33) may have to be modified. (Indeed, the requirement that the wavelengths of the dominant unstable modes be small compared to the radial scale length will probably set a lower limit on the ion current for which this model is valid.) A proper determination of ν_e would require a three-dimensional particle simulation of a contactor cloud and experimental observations in the relevant regime to make sure that the simulation includes all of the relevant physics. Short of that, Eq. (33) is a reasonable guess that should be of help in choosing parameters for more careful theoretical and experimental studies. The method we will use to find analytic expressions for $\phi(r,z)$ and the collected electron current may also be applied using more realistic expressions for ν_e.

The divergence of the radial flux of electrons due to ν_e and the radial electric field and temperature and density gradients must be balanced by an inward flux of electrons along the magnetic field neglecting ionization and recombination:

$$\frac{1}{r}\frac{\partial}{\partial r} rn_e v_r + \frac{\partial}{\partial z} n_e v_z = 0 \qquad (34)$$

At high densities, such as those in the experiment of Stenzel and Urrutia,[25] with $\omega_{pe} \gg \omega_{ce}$, the mean free path of electrons will be short compared to the length of the contactor cloud, and the velocity v_z along the magnetic field may also be found by balancing the force from the electric field $e\partial\phi/\partial z$ with the drag force $m_e\nu_e v_z$. In this case Eq. (34) will generally not be separable in r and Z, and it is necessary to solve a fully two-dimensional partial differential equation. The boundary conditions will be that $v_r = 0$ and $\phi = 0$ at the same surface, and the flux of electrons across this surface must be equal to the flux of the electron saturation current of the ambient plasma (along the magnetic field) outside the surface. The potential $\phi(r,z)$ would be quasineutral everywhere.

Since the position of the $\phi = 0$ surface is not known in advance, this would be a difficult numerical problem. The ambient plasma in LEO has much lower density, $\omega_{pe} \lesssim \omega_{ce}$, and this would also be true in most of a space-based contactor cloud, which, as we will show, would extend along the magnetic field to a distance where the cloud density is comparable to the ambient density. In this case, the electrons will flow freely along the magnetic field, and a different model is needed. If the total potential drop ϕ_0 between the anode and the ambient plasma is greater than T_{ec} and T_{ia}, then double layers will form at a distance z_0 along the magnetic field in

both directions, where

$$J_i = \frac{I_i g(z_0)}{2\pi z_0^2} = \left(\frac{m_e}{m_i}\right)^{1/2} J_e^\infty \qquad (35)$$

for thin double layers, just as in the magnetized collisionless case [see Eq. (18)]. Here $g(z)$ is a factor to take into account that the ions are focused by the potential $\phi(r,z)$ if it is not spherically symmetric. Although the flow of electrons along the magnetic field is nearly collisionless, we will assume that there is enough drag to slow down the incoming electrons slightly so that they will not escape out the other end but will become trapped in the cloud. Only a small amount of drag is needed for this if $\phi_0 \gg T_{ea}$, and this could be provided by electron-electron streaming instabilities, which produce effective collision frequencies of only a small fraction of ω_{pe}. Even if all of the electrons are not trapped, making this assumption will not introduce a large error if most of them are trapped. At $z = \pm z_0$, the flux of electrons along the field must then satisfy the boundary condition

$$n_e v_z = \mp J_e^\infty / e \qquad (36)$$

Because the flow of electrons across the magnetic field is collisional, no double layer exists in the radial direction. For fixed $|z| < z_0$, $\phi(r,z)$ must decrease smoothly to zero at some $r_1(z)$ satisfying quasineutrality all the way. For fixed r, along a given field line, as long as $\phi(r,z = 0) > T_e(r)$, $\phi(r,z)$ will not go to zero for $|z| < z_0$. If ϕ_0 is at least a few times greater than T_e, then $\phi(r,z = 0)$ will be greater than T_e for all r not too close to r_1. It follows that r_1 is nearly independent of z. The contours of $\phi(r,z)$ and the flow of ions and electrons are shown schematically in Fig. 5.

This means that Eq. (34) will be separable in r and z. The boundary conditions in r are

$$\phi(r = r_{\text{anode}}, z) = \phi_0 + T_e \ell n[n_e(z)/n(z = 0)] \qquad (37)$$

$$\phi(r = r_1) = 0 \qquad (38)$$

$$\frac{\partial \phi}{\partial r} = \frac{1}{e} \frac{\partial T_e}{\partial r} \qquad \text{at} \quad r = r_1 \qquad (39)$$

The last condition follows from the fact that $v_r = 0$ outside the contactor cloud, and there is no source or sink of electrons at $r = r_1$; hence v_r must vanish at r_1 just inside the contactor cloud. Equation (31) (with $T_e = 0$) and Eq. (32) then yield Eq. (39).
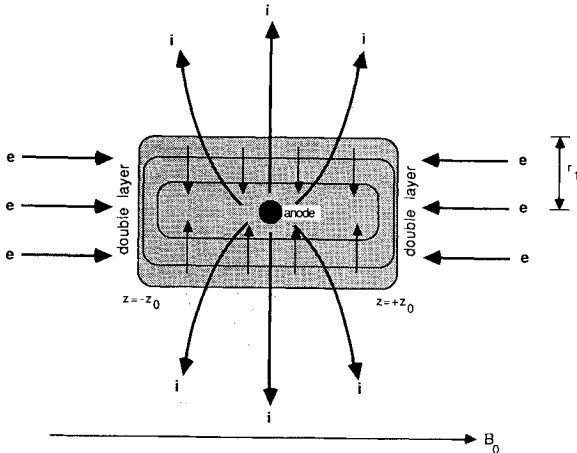

Fig. 5 Schematic picture of the anisotropic contactor model showing equipotential contours and the flow of ions and electrons.

A. Electron Temperature

Before proceeding with the calculation of the potential profile $\phi(r)$, we will briefly consider whether we are justified in assuming that ϕ_0 is at least a few times greater than T_e. The electron temperature profile $T_e(r)$ is determined by the balance among convection, conduction, and ohmic heating (both perpendicular and parallel to the magnetic field). We neglect ionization and line radiation, which should only be important near the anode, and we neglect heat lost by electrons boiling out along the magnetic field.

$$\frac{-3}{2} v_r \frac{\partial T_e}{\partial r} + \frac{1}{rn_e} \frac{\partial}{\partial_r} r\kappa \frac{\partial T_e}{\partial r} + ev_r \frac{\partial \phi}{\partial r} + \frac{J_e^\infty}{n_e z_0}\left(\phi - \frac{T_e}{e}\right) = 0 \quad (40)$$

Here κ is the cross-field thermal conductivity, which is dominated by turbulence just as the drag is. In general

$$\kappa = \frac{Cn_e T_e v_e}{m_e \omega_{ce}^2} \qquad (41)$$

where C is a constant, which depends on the details of the "collisions" causing the heat transport. For electron thermal conductivity across a magnetic field due to Coulomb collisions,[26] for example, $C = 4.7$.

The boundary conditions are

$$T_e = 0 \qquad \text{at} \qquad r = r_1 \qquad (42)$$

$$\kappa = \frac{\partial T_e}{\partial r} = \frac{Q}{4\pi r_{\text{anode}} z_0} - n_e v_r T_e \qquad \text{at} \quad r = r_{\text{anode}} \qquad (43)$$

where Q is the heat flux going into the anode. This is generally greater than the convective heat flux into the anode (the second term on the right side), because $\langle v_x^3 \rangle$ for a half Maxwellian is greater than $\langle v_x^2 \rangle \langle v_x \rangle$. So $\partial T_e / \partial r > 0$ at r_{anode}. Because $T_e = 0$ at $r = r_1$, $\partial T_e / \partial r$ must change sign between r_{anode} and r_1, and we can estimate that the second term in Eq. (40) is of order $-\kappa T_e / n_e r_1^2$. Using Eqs. (31), (32), and (41) we find

$$v_r = \frac{\kappa}{n_e T_e C}\left(e\frac{\partial \phi}{\partial r} - \frac{\partial T_e}{\partial r}\right) \qquad (44)$$

Then the first term in Eq. (40) is of order $\pm \kappa e\phi / Cn_e r_1^2$, and the third term is of order $+\kappa e^2\phi^2 / Cn_e T_e r_1^2$. From Eqs. (34) and (36), the fourth term in Eq. (40) is comparable to (and has the same sign as) the third term.

If $C \leq 1$, it follows that the second and/or the first term must balance the third and fourth terms; therefore T_e is of order $e\phi$. If $C \gg 1$, then the second term alone must balance the third and fourth terms, and $T_e \approx e\phi/C^{1/2} \ll e\phi$. Our assumption that T_e is at least a few times less than ϕ is thus valid if C is somewhat greater than 1. This is true for Coulomb collisions; whether it is true for ion acoustic turbulence is an open question that is beyond the scope of this paper. If κ is dominated by an energetic tail of the electron distribution, perhaps electrons collected from the ambient plasma which have not yet thermalized, then $C \gg 1$.

B. Potential Profile and Cloud Radius

To find $\phi(r)$, we first integrate Eq. (34) over z from $-z_0$ to $+z_0$ and use Eq. (36) to eliminate v_z

$$\int_{-z_0}^{+z_0} dz \frac{1}{r} \frac{\partial}{\partial r} rn_e v_r = 2J_e^\infty \qquad (45)$$

(It may seem counterintuitive that finding the electron current should require an integration over z since no electrons are collected across field lines at the boundary of the cloud at $r = r_1$, only along field lines at the boundary at $z = \pm z_0$. The purpose of the z integration is simply to show that the dominant contribution to the cross-field electron transport comes

from small z so that ν_e may be evaluated at $z = 0$ when the radial integration is done.) To obtain an expression for n_e, which appears explicitly in Eq. (45) and also implicitly through the dependence of ν_e on ω_{pe}, use quasineutrality

$$n_e = n_i = (4\pi)^{-1} I_i m_i^{1/2} e^{-3/2} (r^2 + z^2)^{-1} g(r,z)(\phi_0 - \phi)^{-1/2} \quad (46)$$

The expression for n_i in Eq. (46) comes from the fact that the ions are unmagnetized and expanding spherically from the anode. The factor $g(r,z)$ takes into account the focusing of the ions by $\phi(r,z)$, which is not spherically symmetric. Using Eq. (32) for ν_r, Eq. (33) for ν_e, Eq. (46) for n_e, taking $B_0 = 0.3$ G, defining the ion atomic weight $\mu = m_i/m_p$, and expressing I_i in amps, J_e^∞ in amps/m^2, and ϕ and ϕ_0 in volts, Eq. (45) becomes

$$\int_{-z_0}^{+z_0} dz \frac{1}{r} \frac{\partial}{\partial r} \left[r(\phi_0 - \phi)^{-3/4} (r^2 + z^2)^{-3/2} g(r,z) \frac{\partial \phi}{\partial r} \right]$$

$$= -12 I_i^{-3/2} \mu^{-3/4} J_e^\infty \quad (47)$$

Because $(\phi_0 - \phi)$ and $\partial\phi/\partial r$ are fairly independent of z and the integrand is most strongly weighted near $z = 0$, we replace ϕ and $\partial\phi/\partial r$ by their values at $z = 0$; therefore they can be taken out of the integral. Similarly, we can set $g(r,z) \approx 1$ because self consistently there cannot be a strong focusing effect for $z < r$ where most of the contribution to the integral is. We then do the integration over z

$$\frac{\partial}{\partial r} \left[\frac{1}{r} (\phi_0 - \phi)^{-3/4} \frac{\partial \phi}{\partial r} \right] = -12 r I_i^{-3/2} \mu^{-3/4} J_e^\infty \quad (48)$$

We integrate Eq. (48) over r using the boundary condition Eq. (39) to obtain the integration constant

$$\frac{1}{r} (\phi_0 - \phi)^{-3/4} \frac{\partial \phi}{\partial r} = 6 I_i^{-3/2} \mu^{-3/4} J_e^\infty (r_2^2 - r^2) \quad (49)$$

where

$$r_2^2 = r_1^2 + \frac{1}{6} r_1^{-1} \phi_0^{-3/4} e^{-1} \frac{\partial T_e}{\partial r} I_i^{3/2} \mu^{3/4} (J_e^\infty)^{-1} \quad (50)$$

We integrate over r again, using Eq. (37) at $z = 0$ to obtain the integration constant

$$(\phi_0 - \phi)^{1/4} = 0.5 I_i^{-3/2} \mu^{-3/4} J_e^\infty (2r_2^2 r^2 - r^4) \quad (51)$$

Finally we use Eq. (38) in Eq. (51) to obtain an equation for r_1

$$\phi_0^{1/4} = 0.5 I_i^{-3/2} \mu^{-3/4} J_e^\infty$$

$$\times \left[r_1^4 + \frac{1}{3} r_1 \phi_0^{-3/4} e^{-1} \frac{\partial T_e}{\partial r} I_i^{3/2} \mu^{3/4} (J_e^\infty)^{-1} \right] \quad (52)$$

If, as we have been assuming, $T_e \ll e\phi_0$, then the second term in brackets may be neglected, and

$$r_1 = 1.2 \phi_0^{1/16} I_i^{3/8} \mu^{3/16} (J_e^\infty)^{-1/4} \quad (53)$$

Note that r_1 has an extremely weak dependence on ϕ_0. For almost any reasonable ϕ_0, say 10 V $< \phi_0 < 1000$ V, for argon and for $J_e^\infty = 2$ mA/m^2, which is between the typical dayside and nightside values

$$r_1 \approx 15 I_i^{3/8} \quad (54)$$

and

$$I_e = 2\pi r_1^2 J_e^\infty \approx 2 I_i^{3/4} \quad (55)$$

In general the total current $I = I_i + I_e$ is

$$I = I_i + 8(J_e^\infty)^{1/2} I_i^{3/4} \mu^{3/8} \phi_0^{1/8} \quad (56)$$

A substantial ambient electron current can be collected for values of ϕ_0 and total current that are of interest for tethers. For 1 A of argon at J_e^∞ 2 mA/m^2, for example, we get a gain $I/I_i = 3$, whereas for 0.5 A of xenon, at a typical dayside electron saturation current $J_e^\infty = 20$ mA/m^2, we obtain $I/I_i = 12$. These gains, although not as large as the gains that were found with a completely collisionless double-layer model, can still make a significant contribution to operation of tethers for power generation. These electron currents are much greater than the electron currents found in the quasineutral model of Ref. 4; the physical reason for this is that electrons are transported across the magnetic field from much greater radius where $\nu_e \ll \omega_{ce}$.

In Fig. 3, the total current is shown for a fixed ion current of 1 A, as a function of electron saturation current, using Eq. (56) and is compared to the total current for the isotropic quasineutral model discussed in Sec. III and for the collisionless, double-layer model using an ion current of 0.01 A. Note that the current from Eq. (56) is much more sensitive to the electron saturation current than in the case of the collisionless, double-layer model. The reason is that the anisotropic contactor cloud, unlike the collisionless, double-layer cloud, cannot easily expand to a larger radius to make up for a decrease in the ambient electron density. In Fig. 4, the current voltage characteristic is shown from Eq. (56) for $J_e^\infty = 2$ mA/m^2 and compared to the results from the isotropic quasineutral model and from the collisionless, double-layer model for a range of electron saturation currents. For realistic potentials, less than 1000 V, the current from Eq. (56) is at least an order of magnitude greater than for the collisionless, double-layer model.

Table 2 shows the load power P_{load} against efficiency using the same ambient plasma and tether parameters as in Table 1 but using Eq. (56) to relate I and ϕ_0. In this case, the maximum power obtained at $\approx 80\%$ efficiency is 12 kW, much higher than in Table 1. Of course in a comparison with the collisionless, double-layer results, the energetic cost of producing more ion current must be compared to the cost of the high potential associated with the space-charge-limited double layer.

Finally in Table 3 we show the power to the load for a quasineutral model, which just emits an ion beam or a double-layer model with ionization so that a large current flows for very low potential drop ($\Delta\phi \approx 0$, $\xi = 1$). At 90% efficiency, this configuration, which makes no use of the ambient plasma, can generate only slightly higher power than the anisotropic contactor and requires substantially higher emitted ion current. This shows that the anisotropic contactor could make a significant contribution to the operation of tethers for power generation.

Table 2 Load power against efficiency of anisotropic contactor

η	I_i(A)	ξ	I(A)	P_{load}(kW)
0.1	1	4.99	4.99	2.64
0.3	1	4.83	4.83	7.65
0.5	1	4.6	4.6	12.1
0.7	1	4.22	4.22	15.4
0.9	1	2.32	2.32	10.6

Table 3 Load power against efficiency of emitting an ion beam

η	I_i(A)	ξ	I(A)	P_{load}(kW)
0.1	22.64	1	22.64	12.1
0.3	17.56	1	17.56	28.1
0.5	12.48	1	12.48	33.3
0.7	7.4	1	7.4	27.7
0.9	2.3	1	2.3	11.2

Table 4 Contactor models and the regimes where they are valid

Model	Limits of validity	Applicable situations	Where discussed
Bare ion source	$I_i > I_e$	$I_i > 1$ A in nightside LEO	Table 3
Bare anode	$I_i < I_e$ $r_{outer} < 2r_{anode}$	Very low I_i, large r_{anode} e.g., $I_i < 10 \mu A$, $r_{anode} = 1$ m, LEO	Briefly in Sec. II
Collisionless unmagnetized double layer	$r_{outer} > 2r_{anode}$, $r_{anode} < r_{inner}$ $r_{outer}^2/r_{anode} < (e\phi_0/m_e)^{1/2}\omega_{ce}^{-1}$ $r_{outer}^2/r_{anode} < (e\phi_0/m_e)^{1/2}v_e^{-1}$	$1 \mu A < I_i < 1$ mA in dayside LEO ground-based experiments[19] with $I_i < 1$ A	Sec. III
Low collisionality magnetized	$r_{outer} > 2r_{anode}$ $r_{outer}^2/r_{anode} > (e\phi_0/m_e)^{1/2}\omega_{ce}^{-1}$ $v_e < \omega_{ce}$, $k_\perp r_1 > 1$	1 mA $< I_i < 100$ mA? in dayside LEO	Not discussed
Anisotropic (magnetized collisional)	$v_e < \omega_{ce}$ at r_1 $\rho_e < r_1$, $k_\perp r_1 > 1$ $r_{anode} < r_1$, $I_i < I_e$	$I_i = 1$ A in dayside LEO	Sec. IV
Isotropic (unmagnetized collisional)	$v_e > \omega_{ce}$ at r_1 $v_e/v_e < r_1$ $k_{1\perp} r_1 > 1$, $I_i < I_e$	Marginally in experiment of Stenzel and Urrutia[25] never in LEO	Briefly in Sec. 4 similar model in Ref. 4

IV. Conclusions

We have examined several models for electron collection by plasma contactors. The range of validity of the different models and the situations where they are applicable are summarized in Table 4.

The ground-based experiments at currents below 1 A appear to be well described by a double-layer model that treats the electrons as collisionless and unmagnetized. In those experiments, the double layer forms approximately at the radius where the plasma emitted from the contactor reaches the ambient plasma density. This radius is less than or comparable to both the electron Larmor radius and the mean-free path of the electrons, based on a model for effective collisions due to instabilities. In high-power space applications, where the plasma cloud must have a radius of tens of meters and the ambient electron Larmor radius is only a few centimeters, neither of these conditions applies. Still neglecting collisions, but taking into account the finite-electron Larmor radius, we find that ambient electrons can get across the double layer and reach the anode only if the Parker-Murphy condition[11] is satisfied (and even that is not a sufficient condition). For $r_{anode} \ll r_{outer}$ and ion current greater than the right side of Eq. (24) (about 1 mA for dayside LEO, even lower for nightside), the Parker-Murphy condition cannot be satisfied for a spherically symmetric double layer with space-charge limited current since the r_{inner} determined by Wei and Wilbur[9] would be less than r_{anode}, for any potential and r_{outer} satisfying the Parker-Murphy condition. This means that such collisionless double layers are not possible in space except at very low ion currents. This conclusion follows from the mass ratio and the magnetic field, electron density, and temperature found in LEO [since the right side of Eq. (24) depends only on these parameters] and does not depend on any assumption made about the potential or the size of the anode other than $r_{anode} \ll r_{outer}$. (Collisionless double layers with higher ion currents are possible if r_{anode} is made big enough so that the bare anode could collect almost as much electron current as the contactor cloud, but the contactor cloud would then serve no purpose.) At higher ion currents and small anodes, if we assume the electrons are still collisionless, the collected electron current will not be space-charge limited, as assumed by Wei and Wilbur, but will be limited to a lower value by the magnetic field. Neglecting the requirement that $r_{inner} > r_{anode}$ and considering only the Parker-Murphy condition, we found an upper limit to the collisionless electron current that could be collected and a lower limit to the potential as a function of ion current. We found that such a large potential is needed across the double layer in order to draw a reasonably large electron current that the available load power for a 20-km long tether is never greater than 400 W. The maximum power is surely far less than this since this figure was found for a configuration with

$r_{inner} \ll r_{outer}$, and the Parker-Murphy condition is known to be far from sufficient in that limit; also, $r_{inner} > r_{anode}$ was known to be far from satisfied at the maximum power. The collisionless double-layer model should be valid in space for emitted ion current sufficiently low ($I_i < 1$ mA for dayside LEO, much lower for nightside) that a double layer can form with $\phi_0 < 5$ kV (the total tether voltage) allowing electrons to get across the magnetic field to the anode and satisfying $r_{inner} > r_{anode}$. There is a further requirement for validity: the electrons must not be deflected from the anode by effective collisions, due to instabilities, as they are traversing the contactor. But this requirement is easily satisfied in space where the ambient ω_{pe} is not too much greater than ω_{ce}.

Since the plasma contactor described by the collisionless, double-layer model cannot generate anything close to the desired power, we must use much higher emitted ion currents. Although the transition from the collisionless, double-layer model to the collisional, quasineutral model is not completely understood, we expect at sufficiently high ion current that there will be instabilities strong enough to produce a high effective electron collision frequency in the contactor cloud. Such a contactor can be described by a collisional quasineutral fluid model in which electrons can flow across the magnetic field within a radius r_{core} of the anode. If r_{core} is defined conservatively as the radius within which the effective electron collision frequency, due to ion acoustic and Buneman instabilities, exceeds the electron cyclotron frequency,[14] then we find that the contactor has a very low impedance but draws very little electron current because r_{core} is rather small. The total contactor current is hardly enhanced at all above the ion current that it is emitting. Even for those cases of higher T_e where a modest gain in current occurs, that gain is due almost entirely to ionization of neutral gas emitted by the contactor, not to collection of electrons from the ambient plasma. In this case, the gas would probably be used more efficiently if it were ionized internally in an ion source rather than externally where much of it can be lost.

If we include the anisotropic part of the contactor cloud where the effective electron collision frequency is less than the electron cyclotron frequency, then electrons can be collected out to a much larger radius, and an electron current a few times greater than the ion current can be drawn from the ambient plasma, even at fairly low potentials. In contrast to the upper limits derived for the collisionless, double-layer model and to the quasineutral model based on the more conservative definition of r_{core}, the electron current has a significant dependence on the electron saturation current of the ambient plasma in this case and is substantially higher for a given ion current on the dayside than on the nightside in equatorial LEO. Analytic expressions for the potential profile and collected electron current can be obtained when the elec-

tron motion along the magnetic field is fairly collisionless so that a double-layer forms in that direction, but the electrons flow collisionally across the magnetic field. This is the regime that is relevant to high current plasma contactors in LEO. Although the model, which is solved analytically in Sec. III, made the simple approximation that the effective electron collision frequency, due only to ion acoustic turbulence, is equal to $10^{-2}\omega_{pe}$, independent of T_e and the electric field, the same method should be applicable using more realistic expressions for the effective collision frequency. Another approximation made in our analysis of this model is that there is sufficient electron thermal conductivity across the magnetic field to keep T_e much lower than ϕ_0 in the contactor cloud. The validity of this approximation must be examined using realistic turbulence models. If this approximation is at least marginally valid, then our results should be qualitatively correct.

One important conclusion of our analysis is that most of the present ground-based experiments have limited relevance to space applications of plasma contactors since they operate in a regime where the magnetic field and effective collisions are not important, or only marginally important. This is true of space-based contactors only at very low current and power levels. An exception is the experiment of Stenzel and Urrutia,[25] which examined a plasma in which the electron Larmor radius was small compared to the scale of the potential, and anomalous transport of electrons across the magnetic field was important. Indeed, they found that the anode collected an electron current a few times greater than the saturation current of the flux tube that intersected the anode, even when the effective collision frequency was less than the electron cyclotron frequency. Urrutia and Stenzel attributed their cross-field electron transport to ion acoustic instabilities that were excited by the azimuthal $E \times B$ drift of the electrons relative to the unmagnetized ions, which gave rise to azimuthal wave electric fields that cause radial $E \times B$ drifts. In this respect the experiment was similar to the anisotropic contractor cloud model considered in Sec. III. However, this experiment differed in one important respect from the regime, appropriate to LEO, that was considered in Sec. III. In the experiment, the density was about 2×10^{11} cm^{-3} and $\omega_{pe}/\omega_{ce} \approx 50$, much higher than in LEO, and as a result, the anomalous parallel resistivity, due to Buneman and ion acoustic instabilities excited by the relative electron and ion flow velocity along the magnetic field, was high. The electrons did not flow freely along the magnetic field but diffused along the field like a collisional fluid, so there were no double layers along the field. It would be desirable to do ground-based experiments in the regime where the electrons flow freely along the magnetic field but collisionally across the magnetic field, since this is applicable to high-power plasma contactors in LEO, and to compare the measured $\phi(r,z)$ and collected current to the expressions calculated in Sec. III or to similar expressions found with more realistic models for ν_e.

Another interesting feature seen by Urrutia and Stenzel is that the enhanced electron current was not continuous in time but occurred in periodic bursts as the instabilities periodically grew up, saturated, and decayed. This behavior is probably due to the positive bias instability, which has been widely observed in configurations of this sort.[17] It is not known whether similar behavior would occur in the regime of free electron flow along the magnetic field and collisional flow across the magnetic field appropriate for LEO. Theoretical and experimental studies are needed to answer this question, which could have important implications for power systems based on electrodynamic tethers in space.

Acknowledgments

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